Lösungen (ohne Gewähr, Fehler bitte melden!):

	[redacted]					
a)	5 cm	6 cm	7 cm	44,4°	57,1°	78,5°
b)	10 cm	12 cm	15,66 cm	39,7°	50°	90,3°
c)	5 cm	6 cm	n. l.	80°	n. l.	n. l.
d)	5 cm	4 cm	3 cm	90°	53,1°	36,9°
e)	5,58 cm	10,50 cm	11 cm	30°	70°	80°
f)	6,62 cm ~~21,95 cm~~	13 cm	14,59 cm ~~25,51 cm~~	27°	63°	90°
g)	14 cm	7 cm	12,12 cm	90°	30°	60°
h)	38 cm	26,40 cm	27 cm	46,9°	44°	89,1°
i)	32,6 cm	326 mm	32,6 cm	60°	60°	60°
j)	2 m	89 cm	11 cm	n. l.	n. l.	n. l.
k)	700 m	350 m	500 m	109,6°	28,1°	42,3°
l)	3,92 cm 1,28 cm	0,2 cm	3 mm	101,4° 18,6°	30°	48,6° 131,4°
m)	n. e. l.	n. e. l.	n. e. l.	23°	87°	70°
n)	84,69 cm	90 cm	35,21 cm	23°	87°	70°
o)	90 cm	230,02 cm	216,45 cm	23°	87°	70°
p)	0,53 m	0,56 m	0,06 m	56°	118,4°	5,6°
q)	123 mm	12,3 cm	1,23 dm	60°	60°	60°
r)	3,14 km	3,14 km	3,14 km	60°	60°	60°
s)	66,10 km	65,65 km	546 m	145,73°	34°	0,27°
t)	32 cm	46 cm	14,02 cm	2,3°	176,7°	1°
u)	471,24 dm 5,69 dm	46 m	463 dm	61,4° 0,6°	59°	59,6° 120,4°
v)	n. l.	46 m	540 dm	n. l.	59°	n. l.
x)	459,21 km	48 km	456,69 km	90°	6°	84°
y)	1,467 km	2,345 km	3,770 km	6,9°	11°	162,4°
z)	144,03 cm	123 cm	189,40 cm	57°	33°	90°

a) $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$\cos(\alpha) = \dfrac{a^2-b^2-c^2}{-2bc} = \dfrac{5^2-6^2-7^2}{-2\cdot 6\cdot 7} = \dfrac{5}{7} \quad |\cos^{-1}$

$\alpha = 44{,}4°$

$\cos(\beta) = \dfrac{b^2-a^2-c^2}{-2ac} = \dfrac{6^2-5^2-7^2}{-2\cdot 5\cdot 7} = \dfrac{19}{35} \quad |\cos^{-1}$

$\beta = 57{,}1°$

$\gamma = 180° - 44{,}4° - 57{,}1° = 78{,}5°$

b) $\dfrac{\sin(\alpha)}{a} = \dfrac{\sin(\beta)}{b}$

$\sin(\alpha) = \dfrac{\sin(\beta)}{b}\cdot a = \dfrac{\sin(50°)}{12}\cdot 10 \approx 0{,}638\ldots \quad |\sin^{-1}$

$\alpha = 39{,}7°$

$\gamma = 180° - 39{,}7° - 50° = 90{,}3°$

$c^2 = a^2 + b^2 - 2ab\cos(\gamma) = 10^2 + 12^2 - 2\cdot 10\cdot 12\cdot\cos(90{,}3°)$

$= 245{,}2566\ldots \quad |\sqrt{\ }$

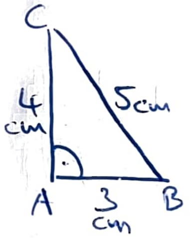
$c = 15{,}66$ cm

c) $\dfrac{\sin(\beta)}{b} = \dfrac{\sin(\alpha)}{a} \quad |\cdot b$

$\sin(\beta) = \dfrac{\sin(\alpha)}{a}\cdot b = \dfrac{\sin(80°)}{5}\cdot 6 = 1{,}18\ldots \quad |\sin^{-1}$

nicht möglich!

d) $3^2 + 4^2 = 5^2$ $(9+16=25!)$



$\Rightarrow \alpha = 90°$

$\sin(\beta) = \dfrac{b}{a} = \dfrac{4}{5} = 0{,}8$

$\Rightarrow \beta = 53{,}1°$

$\Rightarrow \gamma = 36{,}9°$

Alternative:

$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$\cos(\alpha) = \dfrac{a^2-b^2-c^2}{-2bc} = \dfrac{5^2-4^2-3^2}{-2\cdot 4\cdot 3}$

$\cos(\alpha) = 0 \quad |\cos^{-1}$

$\alpha = 90°$

e) $\gamma = 180° - 30° - 70° = 80°$

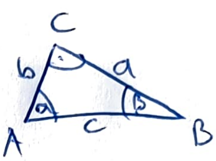
$\frac{a}{\sin(\alpha)} = \frac{c}{\sin(\gamma)} \quad | \cdot \sin(\alpha)$

$a = \frac{c}{\sin(\gamma)} \cdot \sin(\alpha) = \frac{11}{\sin(80°)} \cdot \sin(30°) = 5{,}58\,cm$

$\frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} \quad | \cdot \sin(\beta)$

$b = \frac{c}{\sin(\gamma)} \cdot \sin(\beta) = \frac{11}{\sin(80°)} \cdot \sin(70°) = 10{,}50\,cm$

f) $\gamma = 180° - 63° - 27° = 90°$



$\overset{\cos}{\cancel{\tan}}(\alpha) = \frac{b}{c} \quad | \cdot c$

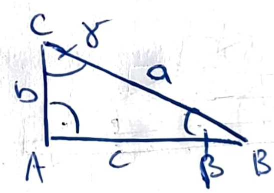
$b = c \cdot \tan(\alpha) \quad | : \overset{\cos}{\cancel{\tan}}(\alpha)$

$c = \frac{b}{\underset{\cos}{\cancel{\tan}}(\alpha)} = \frac{13}{\underset{\cos}{\cancel{\tan}}(27°)} = \underset{14{,}59}{\cancel{25{,}51}}\,cm$

$a^2 + b^2 = c^2 \quad | - b^2$

$a^2 = c^2 - b^2 = \overset{14{,}59}{\cancel{25{,}51}}{}^2 - 13^2 = \overset{43{,}86\ldots}{\cancel{481{,}96}}\ldots \quad | \sqrt{\ }$

$a = \overset{6{,}62}{\cancel{21{,}95}}\,cm$



g) $\beta = 180° - 90° - 60° = 30°$

$\sin(\beta) = \frac{b}{a} \quad | \cdot a$

$b = a \cdot \sin(\beta) = 14 \cdot \sin(30°) = 7\,cm$

$b^2 + c^2 = a^2 \quad | - b^2$

$c^2 = a^2 - b^2 = 14^2 - 7^2 = 147 \quad | \sqrt{\ }$

$c = 12{,}12\,cm$

h) $b^2 = a^2 + c^2 - 2ac\cos(\beta)$

$b^2 = 38^2 + 27^2 - 2 \cdot 38 \cdot 27 \cdot \cos(44°) = 696{,}91... \quad |\sqrt{\ }$

$b = 26{,}40$ cm

$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} \quad | \cdot a$

$\sin(\alpha) = \frac{\sin(\beta)}{b} \cdot a = \frac{\sin(44°)}{26{,}4} \cdot 38 = 0{,}999887... \quad |\sin^{-1}$

$\alpha = 89{,}1°$

$\gamma = 180° - 44° - 89{,}1° = 46{,}9°$

i) $\frac{\sin(\gamma)}{c} = \frac{\sin(\beta)}{b} \quad | \cdot c$

$\sin(\gamma) = \frac{\sin(\beta)}{b} \cdot c = \frac{\sin(60°)}{\cancel{326\,mm}} \cdot \cancel{32{,}6\,cm} = \sin(60°) \quad | \sin^{-1}$

$\gamma = 60°$

$\alpha = 180° - 60° - 60° = 60°$ $\Rightarrow$ alle Winkel gleich

$\Rightarrow$ gleichseitig

$\Rightarrow$ $a = 32{,}6$ cm

j) $b + c = 1\,m < 2\,m = a$ $\Rightarrow$ kein Dreieck!

(Die Katheten sind zu kurz!)

k) $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$\cos(\alpha) = \frac{a^2 - b^2 - c^2}{-2bc} = \frac{700^2 - 350^2 - 500^2}{-2 \cdot 350 \cdot 500} = -\frac{47}{140} \quad |\cos^{-1}$

$\alpha = 109{,}6°$

$\frac{\sin(\beta)}{b} = \frac{\sin(\alpha)}{a} \quad | \cdot b$

$\sin(\beta) = \frac{\sin(\alpha)}{a} \cdot b = \frac{\sin(109{,}6°)}{700} \cdot 350 = 0{,}47098... \quad |\sin^{-1}$

$\beta = 28{,}1°$

$\alpha = 180° - 28{,}1° - 109{,}6° = 42{,}3°$

l) $\frac{\sin(\gamma)}{c} = \frac{\sin(\beta)}{b}$ $\quad | \cdot c$

$\sin(\gamma) = \frac{\sin(\beta)}{b} \cdot c = \frac{\sin(30°)}{2\,mm} \cdot 3m = 0{,}75 \quad | \sin^{-1}$

$\gamma_1 = 48{,}6°$; $\gamma_2 = 180° - 48{,}6° = 131{,}4°$

$\alpha_1 = 180° - 48{,}6° - 30° = 101{,}4°$; $\alpha_2 = 180° - 131{,}4° - 30° = 18{,}6°$

$a_1^2 = b^2 + c^2 - 2bc\cos(\alpha_1)$

$= 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot \cos(101{,}4°)$

$= 15{,}37\ldots \quad | \sqrt{\ }$

$a_1 = 3{,}92\,cm$

$a_2^2 = b^2 + c^2 - 2 \cdot bc \cdot \cos(\alpha_2)$

$= 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot \cos(18{,}6°)$

$= 1{,}626779\ldots \quad | \sqrt{\ }$

$a_2 = 1{,}28\,cm$

m) nicht eindeutig lösbar!

n) $\gamma = 180° - 23° - 87° = 70°$

$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \quad | \cdot \sin(\alpha)$

$a = \frac{b}{\sin(\beta)} \cdot \sin(\alpha) = \frac{90}{\sin(87°)} \cdot \sin(23°) = 35{,}21\,cm$

$\frac{c}{\sin(\gamma)} = \frac{b}{\sin(\beta)} \quad | \cdot \sin(\gamma)$

$c = \frac{b}{\sin(\beta)} \cdot \sin(\gamma) = \frac{90}{\sin(87°)} \cdot \sin(70°) = 84{,}69\,cm$

o) $\gamma = 180° - 87° - 23° = 70°$

$$\frac{b}{\sin(\beta)} = \frac{a}{\sin(\alpha)} \quad | \cdot \sin(\beta)$$

$$b = \frac{a}{\sin(\alpha)} \cdot \sin(\beta) = \frac{90}{\sin(23°)} \cdot \sin(87°) = 230{,}02 \text{ cm}$$

$$c = \frac{a}{\sin(\alpha)} \cdot \sin(\gamma) = \frac{90}{\sin(23°)} \cdot \sin(70°) = 216{,}45 \text{ cm}$$

p) $\beta = 180° - 56° - 5{,}6° = 118{,}4°$

$$a = \frac{b}{\sin(\beta)} \cdot \sin(\alpha) = \frac{0{,}56}{\sin(118{,}4°)} \cdot \sin(5{,}6°) = 0{,}53 \text{ m}$$

$$c = \frac{b}{\sin(\beta)} \cdot \sin(\gamma) = \frac{0{,}56}{\sin(118{,}4°)} \cdot \sin(5{,}6°) = 0{,}06 \text{ m}$$

q) $a = b = c \Rightarrow \alpha = \beta = \gamma = 60°$

r) $\gamma = 180° - \alpha - \beta = 180° - 60° - 60° = 60° \Rightarrow \alpha = \beta = \gamma$

$\Rightarrow a = b = c = 3{,}14$ km

s) $$\frac{\sin(\gamma)}{c} = \frac{\sin(\beta)}{b} \quad | \cdot c$$

$$\sin(\gamma) = \frac{\sin(\beta)}{b} \cdot c = \frac{\sin(34°)}{65650 \text{m}} \cdot 546 \text{m} = 4{,}65... \cdot 10^{-3} \quad | \sin^{-1}$$

$(= 0{,}00465...)$

$\gamma = 0{,}27°$

$\alpha = 180° - 34° - 0{,}27° = 145{,}73°$

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$= 65650^2 + 546^2 - 2 \cdot 65650 \cdot 546 \cdot \cos(145{,}73°)$$

$$= 4369464582 \quad (4 \text{ Mrd.}) \quad | \sqrt{}$$

$a = 66101{,}93 \text{ m} \approx 66{,}10$ km

t) $c^2 = a^2 + b^2 - 2bc\cos(\gamma)$

$c^2 = 32^2 + 46^2 - 2 \cdot 32 \cdot 46 \cdot \cos(1°)$

$c^2 = 196{,}44\ldots \quad | \sqrt{}$

$c = 14{,}02$ cm ← Wer hier auf 14 rundet, hat kein Dreieck mehr!

$\frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c} \quad | \cdot a$

$\sin(\alpha) = \frac{\sin(\gamma)}{c} \cdot a = \frac{\sin(1°)}{14{,}02} \cdot 32 = 0{,}03983\ldots \quad | \sin^{-1}$

$\alpha = 2{,}3°$

$\beta = 180° - 1° - 2{,}3° = 176{,}7°$

u) $\frac{\sin(\gamma)}{c} = \frac{\sin(\beta)}{b} \quad | \cdot c$

$\sin(\gamma) = \frac{\sin(\beta)}{b} \cdot c = \frac{\sin(59°)}{460\,dm} \cdot 463\,dm = 0{,}8627\ldots \quad | \sin^{-1}$

$\gamma_1 = 59{,}6° \; ; \; \gamma_2 = 180° - 59{,}6° = 120{,}4°$

$\alpha_1 = 180° - 59° - 59{,}6° = 61{,}4° \; ; \; \alpha_2 = 180° - 59° - 120{,}4° = 0{,}6°$

$a_1^2 = b^2 + c^2 - 2bc\cos(\alpha_1) = 460^2 + 463^2 - 2 \cdot 460 \cdot 463 \cdot \cos(61{,}4°)$

$a_1^2 = 222065{,}4162 \quad | \sqrt{}$

$a_1 = 471{,}24$ dm

$a_2^2 = b^2 + c^2 - 2bc\cos(\alpha_2) = 460^2 + 463^2 - 2 \cdot 460 \cdot 463 \cdot \cos(0{,}6°)$

$a_2^2 = 32{,}355\ldots \quad | \sqrt{}$

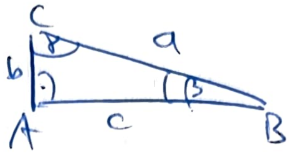
$a_2 = 5{,}69$ dm

v) $\frac{\sin(\gamma)}{c} = \frac{\sin(\beta)}{b} \quad | \cdot c$

$\sin(\gamma) = \frac{\sin(\beta)}{b} \cdot c = \frac{\sin(59°)}{460\,dm} \cdot 540\,dm = 1{,}00623...$

$\Rightarrow$ nicht lösbar!

x) $\beta = 180° - 90° - 84° = 6°$



$\tan(\gamma) = \frac{c}{b} \quad | \cdot b$

$c = b \cdot \tan(\gamma) = 48 \cdot \tan(84°) = 456{,}69\,km$

$a^2 = b^2 + c^2 = 48^2 + 456{,}69^2 = 210869{,}2937 \quad | \sqrt{\ }$

$a = 459{,}21\,km$

y) $\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} \quad | \cdot a$

$\sin(\alpha) = \frac{\sin(\beta)}{b} \cdot a = \frac{\sin(11°)}{2{,}345\,km} \cdot 1{,}467\,km = 0{,}119... \quad | \sin^{-1}$

$\alpha = 6{,}9°$

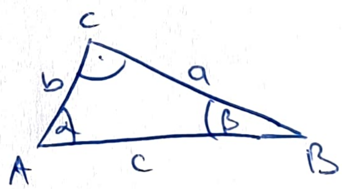
$\gamma = 180° - 6{,}9° - 11° = 162{,}4°$

$c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma) = 1{,}467^2 + 2{,}345^2 - 2 \cdot 1{,}467 \cdot 2{,}345 \cdot \cos(162{,}4°)$

$= 14{,}209... \quad | \sqrt{\ }$

$c = 3{,}770\,km$

z) $\beta = 180° - 90° - 57° = 33°$



$\tan(\alpha) = \frac{a}{b} \quad | \cdot b$

$a = b \cdot \tan(\alpha) = 123\,cm \cdot \tan(57°) = 189{,}40\,cm$

$a^2 + b^2 = c^2 \quad | -b^2$

$a^2 = c^2 - b^2 = 189{,}40^2 - 123^2 = 20744{,}644... \quad | \sqrt{\ }$

$a = 144{,}03\,cm$